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## THE FRACTAL KINETICS OF POLYMERIZATION CATALYZED BY NANOFILLERS

*Keywords: polymerization, kinetics, catalyst, nanofiller, fractal analysis, strange diffusion.*

*The fractal analysis of polymerization kinetics in nanofiller presence was performed. The influence of catalyst structural features on chemical reaction course was shown. The notions of strange (anomalous) diffusion conception was applied for polymerization reactions description.*

*Ключевые слова: полимеризация, кинетика, катализатор, нанонаполнитель, фрактальный анализ, аномальная диффузия.*

*Выполнен фрактальный анализ кинетики полимеризации в присутствии нанонаполнителя. Показано влияние структурных особенностей катализатора на ход химической реакции. Понятие странной (аномальной) диффузии использовано для описания реакций полимеризации.*

### Introduction

By Sergeev's definition the nanochemistry is a science field connected with obtaining and studying of physical-chemical properties of particles having sizes of nanometer scale. Let's note that according to this definition polymers synthesis is automatically a nanochemistry part as far as according to the Melikhov's classification polymeric macromolecules (more precisely macromolecular coils) belong to nanoparticles and polymeric sols and gels – to nanosystems. Catalysis on nanoparticles is one of the most important sections of nanochemistry.

The majority of catalytic systems are nanosystems. At heterogeneous catalysis the active substance is tried to deposit on carrier in nanoparticles form in order to increase their specific surface. At homogeneous catalysis active substance molecules have often in themselves nanometer sizes. The most favourable conditions for homogeneous catalysis are created when reagent molecules are adsorbed rapidly by nanoparticles and are desorbed slowly but have high surface mobility and, consequently, high reaction rate on the surface and at the reaction molecules of such structure are formed at which desorption rate is increased sharply. If these conditions are realized in nanosystem with larger probability than in macrosystem, then nanocatalyst has the raising activity that was observed for many systems. In the connection such questions arise as adsorption and desorption rate, surface mobility of molecules and characteristics frequency of reagents interaction acts depend on the size, molecular relief and composition of nanoparticles and the carrier.

The presence paper purpose is the application of fractal analysis for description of polymerization kinetics in nanofiller presence.

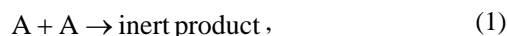
### Results and Discussion

The analysis of polymerization processes in nanofiller presence does not differ principally from the one for transesterification model reaction [1]. In the present chapter some important aspects of such polymerization will be studied, mainly on the example of solid-phase imidization.

The authors [2] studied the kinetics of poly(amic acid) (PAA) solid-state imidization in nanofiller

(Na<sup>+</sup>-montmorillonite) presence and found an essential reaction acceleration both at imidization temperature  $T_i$  raising in the range 423-523K and Na<sup>+</sup>-montmorillonite contents  $W_c$  increase within range 0-7 phr. In paper [2] the possible chemical mechanism of Na<sup>+</sup>-montmorillonite action as a catalyst was offered, assuming larger contact surface area and respectively larger number of reaction active sites, that promotes dehydration and imide ring closure reaction. This model has hypothetical and qualitative character. However, it became obvious lately, that on chemical reactions kinetics large influence can be exercised by purely physical factors such as reactionary medium connectivity [3] or macromolecular coil structure [4], which in all polymer's states (solution, melt, solid phase) is fractal [5, 6]. It is also well known [7], that the fractal objects description is correct only within the framework of fractal geometry and the usage in such case of Euclidean geometry gives approximation more or less corresponding to reality. Proceed from the said above the authors [8, 9] exercised the solid-state imidization reaction description both in nanofiller presence and in its absence within the framework of structural (fractal) models.

In the general and the simplest form solid-state imidization reaction can be represented by the equation [3]:



where A is a reagent (in considered case PAA).

Then for such reaction description the following relationship was used [3]:

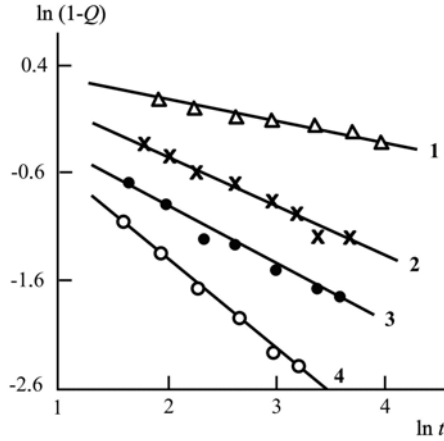
$$\rho_A \sim t^{-d_s/2}, \quad (2)$$

where  $\rho_A$  is the concentration of nonreacted reagent A, which further will be accepted equal to (1-Q) (Q is a conversion degree),  $t$  is a reaction duration,  $d_s$  is a spectral dimension.

In Fig. 1 the dependences  $\rho_A=(1-Q)$  on  $t$  in log-log coordinates, corresponding to the relationship (2), for imidization reaction without filler at the four indicated above imidization temperatures  $T_i = 423, 473, 503$  and 523K are shown.

As follows from the data of Fig. 1, all the four adduced plots are linear, that allows to determine the value of spectral dimension  $d_s$ . The estimations have shown, that the imidization temperature  $T_i$  raising within the range 423-523K results to  $d_s$  increase from 0.42

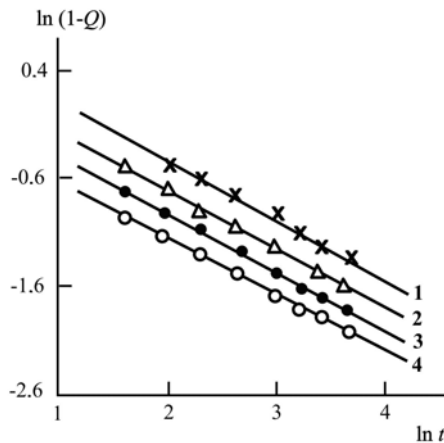
up to 1.68, i.e. to essential growth of reactionary system connectivity degree. In Fig. 2 the similar dependences for various  $\text{Na}^+$ -montmorillonite contents  $W_c$  at fixed  $T_i = 473\text{K}$  are shown. As one can see, nanofiller introduction exercises much weaker influence on  $d_s$  value than the imidization temperature raising [9].



**Fig. 1 - The dependences of  $\rho_A=(1-Q)$  on  $t$  in log-log coordinates, corresponding to the relationship (2), for PAA solid-state imidization without filler at temperatures: 423 (1), 473 (2), 503 (3) and 523 (4) [8]**

$d_s$  increase at  $T_i$  raising for the same reactionary system, shown in Fig. 1, assumes, that in the considered case  $d_s$  should be considered as an effective spectral dimension  $d_s$ , depending on reactionary medium heterogeneity degree [10]. The medium heterogeneity degree can be characterized by heterogeneity exponent  $h$ , which changes within the range  $0 < h < 1$  and turns into zero only for homogeneous samples [10]. The values  $h$  and  $d_s$  are connected with one another by the equation [10]:

$$d_s' = 2(1 - h). \quad (3)$$



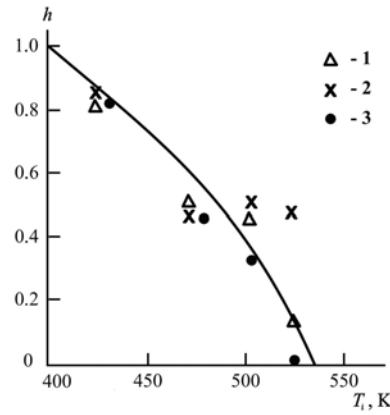
**Fig. 2 - The dependences of  $\rho_A=(1-Q)$  on  $t$  in log-log coordinates, corresponding to the relationship (2), for PAA solid-state imidization at temperature 473 K and  $\text{Na}^+$ -montmorillonite contents  $W_c$ : 0 (1), 2 (2), 5 (3) and 7 (4) phr [8]**

In Fig. 3 the dependence  $h(T_i)$  is shown, from which fast decrease  $h$  or reactionary medium homogeneity raising follows at  $T_i$  increase. At  $T_i \approx 540\text{K}$  the exponent  $h = 0$ , i.e. the reactionary medium becomes homogeneous. The authors [2] have shown, that for the studied polyimides the melting temperature  $T_m$  is equal about to 800K. Proceeding from the known law of two-thirds [11]:

$$\frac{T_g}{T_m} = \frac{2}{3}, \quad (4)$$

the glass transition temperature  $T_g$  of polyimide can be estimated as equal to  $\sim 533\text{K}$ . In other words, as it was expected [12, 13], reactionary medium in solid-state imidization case became homogeneous (Euclidean) at glass transition.

The shape of the curve  $h(T_i)$ , shown in Fig. 3, i.e. tendency for  $h \rightarrow 0$  at temperature raising, assumes, that the fractal-like effects, namely,  $d_s$  variation, are connected with energetic disorder [10].



**Fig. 3 - The dependence of reactionary medium heterogeneity exponent  $h$  on imidization temperature  $T_i$  for PAA solid-state imidization at  $\text{Na}^+$ -montmorillonite contents  $W_c$ : 0 (1), 2 (2) and 5 (3) phr [8]**

Let's consider physical principles of reactionary system connectivity degree change, characterized by effective spectral dimension  $d_s$ , at imidization temperature  $T_i$  and  $\text{Na}^+$ -montmorillonite contents  $W_c$  change. As well as earlier, the value of macromolecular coil fractal dimension  $D_f$  can be estimated with the help of formula [4]:

$$t^{(D_f-1)/2} = \frac{c_1}{k_1(1-Q)}, \quad (5)$$

where  $c_1$  – is constant, determined according to the boundary conditions and accepted equal to  $8 \times 10^{-4} \text{ s}^{-1}$  for studied reactions,  $k_1$  is reaction rate constant.

In Fig. 4 the dependence of  $d_s$  on  $D_f$  is shown, from which it follows, that  $d_s$  increases at  $D_f$  reduction and at  $D_f \rightarrow 1.50$  (transparent macromolecular coil [5]) the value  $d_s$  has a fast tendency to its limiting magnitude  $d_s=2.0$  [10]. Such form of dependence  $d_s(D_f)$  allows to make two conclusions. Firstly, the definite interconnection of  $d_s$  and  $D_f$  characterized by the curve of Fig. 4 exists. Secondly, the value  $d_s$  can not be considered as spectral dimension of proper macromolecular

coil, since in this case theory assumes  $d'_s$  decrease at  $D_f$  reduction [14]:

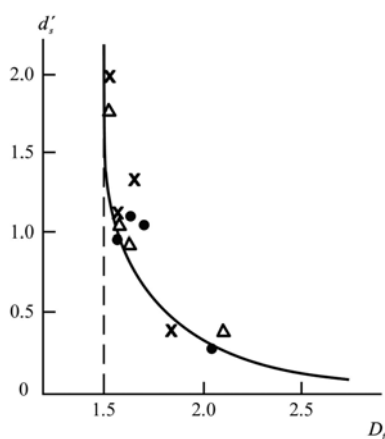
$$d'_s = \frac{2(2D_f - d)}{d + 2}, \quad (6)$$

where  $d$  is dimension of Euclidean space, in which fractal is considered (it is obvious, that in our case  $d = 3$ ).

Proceeding from the said above, let's assume, that in the considered case  $d'_s$  is a reactionary medium connectivity indicator, in some way connected with macromolecular coil structure characterized by dimension  $D_f$ . Let's consider one of the possible theoretical schemes of such interrelation.

It is well known [15], that in chemical reactions large effect has steric factor  $p$  ( $p \leq 1$ ) showing that not all collisions of reagents occur with proper for reaction products formation orientation of reacting molecules. The value  $p$  is defined by dimension  $D_f$  and can be calculated according to the equation [4]:

$$p = \frac{1.6}{10600^{(D_f-1)/2}}. \quad (7)$$



**Fig. 4 - The dependence of reactionary medium effective spectral dimension  $d'_s$  on macromolecular coil fractal dimension  $D_f$  in imidization process. The notation is the same, as in Fig. 3 [8]**

As it follows from this equation, the value  $p$  is increased at  $D_f$  reduction. Macromolecular coil sites number  $N$ , capable to take part in a chemical reaction (active sites), is determined like that [16]:

$$N \sim t^{d'_s/2}. \quad (8)$$

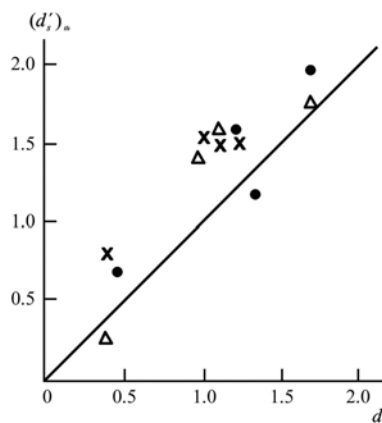
Let's pay attention, that  $N \sim \rho_A^{-1}$  in the relationship (2). But in case of a chemical reaction not all active sites of macromolecular coil can react in virtue of the condition  $p < 1$ , but only their part  $N_p$ , proportional to  $p$ . For preliminary estimations it can be assumed [9]:

$$N_p = 100p. \quad (9)$$

Believing that for imidization reaction without filler at  $T_i = 423K$   $D_f = 2.12$ ,  $d'_s = 0.42$ , i.e. experimentally determined values, one can calculate the value  $p$  according to the equation (7), the value  $N_p$  according to the equation (9) and to determine constant coefficient in the relationship (8) at  $t = \text{const} = 15 \text{ min.}$ , which is equal to 2. Further, using this coefficient in the equation (8), one can calculate the values  $d'_s$ , which further will be designated as  $(d'_s)_{th}$ , for imidization reaction with variable  $T_i$  and  $W_c$ . In Fig. 5 the comparison of values  $d'_s$ , deter-

mined from the slope of plots  $\rho_A(t)$  in log-log coordinates (see Figs. 1 and 2) and  $(d'_s)_{th}$ , calculated according to the equations (7)-(9), is shown. As one can see, between these parameters a good correspondence was obtained. This means, that reactionary space connectivity, characterized by dimension  $d'_s$ , depends on macromolecular coil structure dimension  $D_f$  and this dimension is specific namely for the chemical reactions owing to steric factor  $p$  introduction [9].

From the point of view of process chemistry this can be treated as follows: the smaller  $D_f$ , is the more open macromolecular coil structure, is the easier dehydration process (water removal) and imidic ring closure proceed [2].



**Fig. 5 - The comparison of the calculated according to the relationship (2)  $d'_s$  and the equations (7)-(9)  $(d'_s)_{th}$  effective spectral dimension values of reactionary medium in imidization process. The notation is the same, as in Fig. 3 [8]**

Therefore, the data considered above demonstrated that the main parameter, controlling solid-state imidization rate, is the reactionary system connectivity degree characterized by its effective spectral dimension. In its turn, this dimension is a function of macromolecular coil structure that is polymeric reaction specific feature. Imidization temperature raising defines reactionary medium heterogeneity reduction and corresponding increase of its connectivity degree [8, 9].

In paper [17] the quantitative analysis of the imidization kinetics temperature dependence was given within the framework of one more conception, namely, a chemical reactions kinetics fractal model [4, 18]. The authors [17] have assumed that the cause of imidization reaction acceleration at its temperature  $T_i$  growth is polyimide macromolecular coil structure change, which is the consequence of its molecular characteristic as the last in paper [17] ratio  $C_\infty$  was chosen, which is a polymer chain statistical flexibility indicator [19]. As earlier, the macromolecular coil structure is characterized by its fractal (Hausdorff) dimension  $D_f$ . The interrelation between  $C_\infty$  and  $D_f$  is given by the equation [20]:

$$C_\infty = \frac{D_f}{3(2.28 - D_f)} + \frac{4}{3}. \quad (10)$$

The temperature dependence  $C_\infty$  (and, consequently,  $D_f$ ) can be calculated, using the equation offered in paper [13]:

$$C_{\infty} = \left( 1 - \sqrt{1 - \frac{9 T_m}{8e T_i}} \right) / \text{const}, \quad (11)$$

where  $T_m$  is the melting temperature, for the studied nanocomposites equal to  $\sim 800\text{K}$  [2].

For the constant in the equation (11) determination the following method were used. The general equation of chemical reactions fractal kinetics is the formula [4]:

$$Q = c_0 \eta_0 t^{(3-D_f)/2}, \quad (12)$$

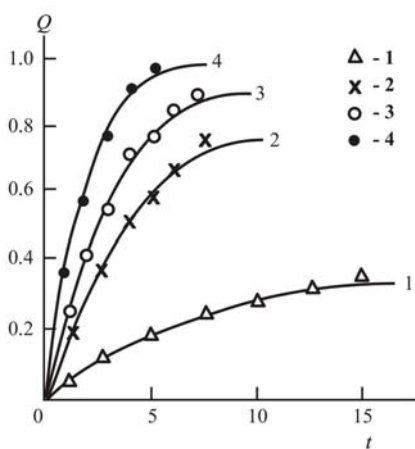
where  $c_0$  is reagent initial concentration,  $\eta_0$  is initial viscosity of reactionary medium.

By constructing the dependence  $Q(t)$  in log-log coordinates at  $T_i = 523\text{K}$  the value  $D_f = 1.59$  was determined, then from the equation (10) corresponding to it the value  $C_{\infty}$  and according to the equation (11) – the value of estimating constant. Further for the remaining imidization temperatures the values  $C_{\infty}$  were calculated (the general variation 3.51-4.51) and corresponding to them the values  $D_f$  (the general variation 1.59-1.81). Then according to the equation (12) the kinetic curves  $Q(t)$  were calculated. The value of reactionary medium initial viscosity  $\eta_0$  in this case was accepted constant and equal to 1 (with taking into account of the fact that the reaction occurs in solid phase) and the value  $c_0$  was determined by the selection method at the condition of the best correspondence of theoretical and experimental curves  $Q(t)$ . Simulation of kinetic curves of solid-state imidization carried out by the considered method is shown by points in Fig. 6. As one can see that a good correspondence of theory and experiment is obtained.

The selection of reagents initial concentration (or reaction active centers) shows it increase within the range 6.5-35 of relative units at imidization temperature  $T_i$  growth within the range 423-523K. This  $c_0$  increase can also be explained within the framework of chemical reactions kinetics fractal conception according to which [4]:

$$M \sim c_0 t_{\text{gen}}, \quad (13)$$

where  $M$  is molecular weight of reaction product,  $t_{\text{gen}}$  is reaction duration.

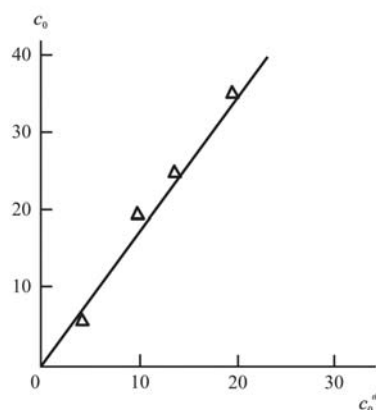


**Fig. 6 - The kinetic curves  $Q(t)$  of solid-state imidization at temperatures: 423 (1, 5), 473 (2, 6), 503 (3, 7) and 523K (4, 8). 1-4 – experimental data; 5-8 – the calculation according to the equation (12) [17]**

Assuming  $M = \text{const}$ , we obtain theoretical value  $c_0(c_0^{\text{th}})$  according to the relationship (13) [17]:

$$c_0^{\text{th}} \sim t_{\text{gen}}^{-1}. \quad (14)$$

Accepting as  $t_{\text{gen}}$  imidization reaction part duration, on which the first order reaction laws are fulfilled and on which imidization mainly was ended, the authors [17] have estimated the values  $c_0^{\text{th}}$ , which are compared with the values  $c_0$ , obtained by a selection method, in Fig. 7. As it follows from the data of this Figure, between the values  $c_0^{\text{th}}$  and  $c_0$  the linear correlation is observed, passing through coordinates origin. Such correspondence assumes, that the value  $c_0$  (imidization active centers number) increases at  $T_i$  raising at the expense of diffusive processes intensification [4].



**Fig. 7 - The relation of by selection obtained  $c_0$  and calculated according to the relationship (14)  $c_0^{\text{th}}$  number of imidization reaction active sites [17]**

Hence, the stated above results have shown, that the fractal conception of chemical reactions kinetic describes quantitatively kinetics of solid-phase imidization process at different temperatures. This description is given only within the framework of reaction physical aspects and does not affect its chemical aspects. The effective initial concentration of reagents  $c_0$  at imidization temperature growth is due to physical cause also – by reagents diffusion intensification in solid-phase state.

It's necessary to indicate that the nanofiller introduction in reactionary mixture results to two-phase system formation, where an important (or decisive) role will be played by interfacial interactions [21]. Particularly, the interaction of PAA- $\text{Na}^+$ -montmorillonite should result to the structure change of polyimide (PI) forming macromolecular coil [22] and similar effect gives the imidization temperature  $T_i$  raising [23]. Therefore the authors [24] fulfilled structural analysis of processes, occurring in solid-phase imidization reaction course, according to the aspect indicated above.

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